Language structure in the *n*-object naming game

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We examine a naming game with two agents trying to establish a common vocabulary for n objects. Such efforts lead to the emergence of language that allows for an efficient communication and exhibits some degree of homonymy and synonymy. Although homonymy reduces the communication efficiency, it seems to be a dynamical trap that persists for a long, and perhaps indefinite, time. On the other hand, synonymy does not reduce the efficiency of communication but appears to be only a transient feature of the language. Thus, in our model the role of synonymy decreases and in the long-time limit it becomes negligible. A similar rareness of synonymy is observed in present natural languages. The role of noise, that distorts the communicated words, is also examined. Although, in general, the noise reduces the communication efficiency, it also regroups the words so that they are more evenly distributed within the available "verbal" space.

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I. INTRODUCTION

Computational modeling is becoming more and more an important tool to study language evolution [1-5]. The central assumption of such an approach is that language is a complex adaptive system that emerges from local interactions between its users and evolves and complexifies according to biological-like principles of evolution and self-organization [6-8]. This is by no means the only possibility since a number of researches claim that language does not have the adaptive values and is merely a by-product of having a large and complex brain or of some other skills [9,10]. Recently, however, adaptationists got strong support from Pinker and Bloom, who in their influential paper [11] argued that linguistic abilities require complex and costly adaptations (e.g., large brain, longer infancy period, and descended larynx) and the language origin can be explained only by means of natural selection theory.

Since language was invented only in one lineage, and is therefore unique to human species, its appearance has the same status as the origin of genetic code or the eukaryotic cell. The emergence of language was thus listed as one of the major transitions in the evolution of life on Earth [12] and it is certainly interesting to ask which factor is responsible for it. Some claims were made that most likely it was the combination of selective evolutionary pressure and unique context that led to the emergence of human language [13].

Language has also led to the novel inheritance system [14] and opened up the possibility for cumulative cultural evolution and creation of complex society [15] with collaboration of large nonkin groups [16]. While our willingness to share information with relatives is rather easy to reconcile with Darwinian evolution (kin selection [17]), linguistic interactions with nonkin individuals are harder to understand. Indeed, since speaking is costly (it takes time, energy, and sometimes might expose a speaker to the predators), and listening is not, such a situation seems to favor selfish individuals that would only listen but would not speak. Moreover, in the case of the conflict of interests the emerging communication system would be prone to misinformation or

lying. A possible resolution of these problems is based on reciprocal altruism [18]. However, there is growing evidence that cooperation and altruistic behavior between humans are very complex and typically cannot be explained using standard reciprocal altruism arguments [16].

As an alternative explanation Dessalles [19] suggests that honest information is given freely because it is profitable—it is a way of competing for status within a group. In this context, interesting computer simulations were made by Hurford [20]. He considered agents engaged in communicative tasks (one speaker and one hearer) and their abilities evolved with the genetic algorithm that was set to prefer either communicative or interpretative success. Only in the former case the emerging language was similar to natural languages where synonymy was rare and homonymy tolerated. When interpretative success was used as the basis of selection then the converse situation (unknown in natural language) arose: homonymy was rare and synonymy tolerated.

Indeed, synonymy in the pure variety is rare. Usually, it can be found in two languages being in contact (napkin/ serviette), handy abbreviations (bicycle/bike) or some specialized euphemistic domains related, e.g., with sex, bodily functions, or death (*die/expire/...*) [20]. Linguists proposed various explanations of the human avoidance of synonymy. Clark attributes it to a presumably inborn tendency of humans to seek and create new meanings, rather than accept one meaning for several different forms [21]. Markman notes that children have a tendency to assume that no two words may overlap in meaning [22]. A similar point of view is expressed in Wexler's uniqueness principle which prevents the child from internalizing more than one form per meaning [23]. On the other hand, homonymy seems to be more common in natural languages. One can easily think of many words having multiple and unrelated meanings (e.g., abstract, compound, second, and present). At first sight one can consider this as surprising since synonymy does not diminish communicative efficiency but homonymy in principle does. Let us also notice that computer languages quite often accept synonymy (e.g., aliases in command systems) but typically do not handle homonymy.

In our opinion an apparent asymmetry between rare synonymy and relatively common homonymy is an important and generic feature of natural languages and might be used as a test of various computational models of language development. In the present paper we examine a version of the Steels naming-game model [25] where two agents exchange information concerning a certain number of facts/objects from their reality and try to establish a common vocabulary. The emerging language features some degree of homonymy and synonymy. Although homonymy diminishes the communicative efficiency it turns out to be a persistent feature of the language. On the other hand, synonymy is only a transient feature of the language and its frequency of appearance diminishes over time. The asymmetry between homonymy and synonymy can be thus understood within a rather simple naming-game setup, without revoking evolutionary arguments that speaker more than hearer benefits from the conversation [20]. Let us also notice that stable homonymy and transient synonymy was also reported by Puglisi et al. in a model of formation of categories [24]. We also examine the role of noise that might distort communicated words. Our results show that noise plays (or played) an important role and could affect the distribution of words in a "verbal" space.

II. MODEL

More than a decade ago Steels proposed the naming-game model, which quickly became one of the basic models of the emergence of linguistic coherence [25]. In this model we have a group of agents that communicate with each other trying to establish a common vocabulary on a certain number of objects. Typically, after some time, they reach a state of linguistic coherence where they to a large extent (or even perfectly) understand each other. In the original formulation the naming-game model describes cultural transmission within a single generation of agents. Evolutionary versions with mutations and selections of agents taking place were also studied [26,27]. In most works on the naming-game model only a simple structure of the emerging language is allowed and homonymy is very often excluded [28-30]. Such approaches effectively can be regarded as if agents would talk on a single object [30]. Although it drastically simplifies the language structure such an approach allows to consider many agents and thus to take into account some elements of the social structure [29]. But such works constitute only one end point of the computational dilemma: many agents of simple architecture versus few but with complex architecture. At the other side we have models of few agents but able to develop language of much larger complexity. To examine linguistic structures such as homonyms or synonyms, one has to consider an n-object version of the naming-game model. Some results on n-object naming-game model have been already reported [31,32]. Let us also notice that the main emphasis in the naming-game model is on the cultural (single-generation) transmission of language. An alternative approach to the language evolution where intergenerational interactions play an important role is called iterated learning model and was used in various contexts [33].

Our model is a two-agent version of the naming game. It is assumed that agents are embodied in a shared environment and communicate on a certain number of facts/objects from this environment. Agents in turns take the role of speaker and hearer. Speaker selects an object from the environment. Then, using its form-meaning relations, speaker selects a word that is assigned to the object. The word is communicated to hearer, which uses its own meaning-form relations to guess the communicated meaning. We also assume that after such a communication attempt there is a possibility to check (e.g., by pointing at the object) whether hearer guessed the communicated meaning correctly. Established in such a way success or failure modifies the structure of meaningform relations of agents to facilitate future communication attempts.

Both agents refer to the common set of n objects and with each object each agent relates the corresponding inventory (inventories are numbered from 1 to n). Each inventory stores up to l words that are used to describe the corresponding object. With each word in a repository the weight w is associated that controls the stochastic process of selecting a communicated word (speaker) and decoding the meaning (hearer). The idea of assigning weights to words was already used in some naming-game models [32]. For computational purposes the words are represented by integer numbers from 1 to r but more natural representations using strings of letters are also possible. The parameter r can be thus interpreted as corresponding to the capacity of the "verbal space." More detailed rules of our model are specified below:

(i) Speaker randomly selects an object. From the inventory that corresponds to the selected object speaker selects the communicated word x_c . The word is selected taking into account the weights corresponding to each word in this inventory. We used the method of roulette selection.

(ii) Hearer tries to guess the meaning of the communicated word and decodes it. To do that, hearer first calculates measures of similarity $s^k(x_c)$ of the communicated word with *k*th inventory (k=1,2,...,n). The measures $s^k(x_c)$, which are calculated using the following formula:

$$s^{k}(x_{c}) = \frac{1}{\sum_{i} w_{i}} \sum_{i} \frac{w_{i}}{\epsilon + |x_{i} - x_{c}|}, \quad k = 1, 2, \dots, n, \quad (1)$$

are then used to select the inventory that fits the communicated word [roulette selection again but with $s^k(x_c)$ as a weight of an inventory]. In Eq. (1) x_i and w_i are the *i*th word and its weight, respectively, and the summation is over all elements of the *k*th inventory (numerated with *i*). The closer x_i to the x_c is, the larger are its contributions to the similarity measure $s^k(x_c)$. The role of ϵ in Eq. (1) is to keep $s^k(x_c)$ finite even when the communicated word is the same as one of the words in the *k*th inventory. Having calculated $s^k(x_c)$ for all inventories, hearer uses the roulette selection to choose the inventory that fits the communicated word. Since in our calculations ϵ takes rather small values, inventories that contain a communicated word (or words that are very close to it) get large similarity measures and have larger probabilities of being selected. (a) When the inventory selected by the hearer has the same number as that selected by speaker, we consider this as a communicative success. In such a case both agents increase the weights associated with the communicated word by one. If in the hearer inventory there is no such a word (but it still has decoded the meaning correctly) we add the communicated word to this inventory with unit weight (if the inventory contains already l elements we first remove the word with the smallest weight).

(b) When the inventory selected by the hearer has a different number than that selected by the speaker we consider this as a communicative failure. In such a case speaker decreases the weight associated with the communicated word by one. Hearer inspects its inventory that has the same number as that selected by the speaker. If it contains the communicated word, its weight is increased. Otherwise, hearer adds the word to this inventory with unit weight.

Our simulations show that the model is relatively robust and small changes in its rules or of values of parameters do not change much the behavior of the model. In particular, similar results are obtained when an increase or decrease in weights in the case of success or failure is done either with a fixed or weight-dependent amount (e.g., the larger the weight, the smaller the increase). Let us also notice that the increase or decrease in weight in the case of success or failure, respectively, resembles the reinforcement learning approach and some naming-game models with a similar dynamics have been already examined [31].

(iii) In some of our simulations we have examined the effect of noise that distorts the communicated word. More precisely, we assume that with the probability p the communicated word chosen by speaker becomes

$$x \to x + \eta, \tag{2}$$

where η is a random integer number uniformly drawn from the interval $\langle -a, a \rangle$ and *a* is the amplitude of noise (with the probability 1-p the communicated word does not change). If *x* calculated using Eq. (2) happens to be outside the range $\langle 1, r \rangle$, a different instance of η is generated.

An example that illustrates the above rules is shown in Fig. 1. Table I collects all parameters of the model.

III. NUMERICAL CALCULATIONS

To start the simulations an initial configuration is needed. We assume that at the beginning each agent has in each inventory a single word (randomly selected from the interval $\langle 1, r \rangle$) with unit weight. To examine the behavior of the model we measured various quantities that in some cases were averaged over certain time intervals or over independent runs. (We define the unit of time as corresponding to 2n communication attempts.) Of particular interest is the communicative success rate of an agent, which is defined as a fraction of successful communication attempts. Some other quantities that allow us to analyze in more details the structure of the emerging language and of the communication process will be specified later.



FIG. 1. A communication attempt with n=4 objects. Speaker selected the second object and one of the words that are associated with this object. The selected word is communicated to the hearer that then decodes its meaning. Since the decoded object is the same as that chosen by the speaker, the above example is considered as a success. The selection of the communicated word and its decoding are stochastic in nature (see the main text) and controlled by weights *w* associated with each word.

A. Basic properties

Simulations show that typically the agents correlate their inventories so that their communication maintains a rather large success rate [Fig. 2(a)]. Of our further interest will be words that in a given inventory have the largest weight. Since some of them might be the same for different inventories, we calculated the number of different largest-weight words in the resulting language. It turned out that this number is close to the number of objects n [Fig. 2(b)] and most of the communication attempts use these largest-weight words [Fig. 2(c)]. It means that in the majority of cases communication between agents proceeds as follows: speaker selects an object and the largest-weight word from the inventory corresponding to this object becomes the communicated word. For small ϵ the similarity measure, as calculated from Eq. (1), is large only for the inventory that contains the communicated word (provided that the weight of this word is not very small). Usually, it happens to be the inventory corresponding to the same object as selected by speaker and thus such an attempt is successful. For larger ϵ (~0.1) the com-

TABLE I. Parameters of the model and ranges of values used in the simulations.

Parameters	Description (values used in simulations)
п	Number of objects $(100 \le n \le 10^3)$
l	Memory size—maximum number of words corresponding to an object $(5 \le l \le 20)$
r	Words—positive integer numbers not greater than $r (500 \le r \le 10^4)$
ε	Ensures that similarity measure in Eq. (1) is finite $(10^{-5} \le \epsilon \le 10^{-1})$
<i>p</i> , <i>a</i>	Parameters describing noise [see Eq. (2)] $(0 \le p \le 0.05, 0 \le a \le 10)$



FIG. 2. The time evolution of (a) the success rate; (b) the number of different largest-weight words; and (c) the fraction of second-largest-weight utterances. Calculations were made for n=500, l=10, and $r=10^3$.

munication between agents deteriorates and both the success rate and the number of different largest-weight words diminish.

There are two factors that contribute to the communication failure. First, it is the finite number of ϵ that implies that similarity measure (1) for different words is positive and thus selection of the inventory made by the hearer might lead to communication failure. Second, homonyms, which as we shall see might appear in our model, also might lead to the communication failure. Their role is discussed in detail in the next subsection.

In Fig. 3 we present the distribution of largest- and second-largest-weight words that is established after a sufficiently long transient. Relatively uniform distribution indicates that these words are uncorrelated. Since some of the second-largest weight words, as discussed below, might be considered as synonyms (of the largest-weight words), the lack of correlations agrees with the observation that synonyms in natural languages are not similar to each other.

B. Homonymy and synonymy

Since agents communicate on more than one object the resulting language might contain homonyms and synonyms.



FIG. 3. The distribution of largest-weight words and the largestweight words after simulations of time $t=10^3$. Calculations were made for n=500, $r=10^3$, $\epsilon=10^{-5}$, and l=10. Different plotting symbols (circles; crosses) correspond to different agents. Quite often both agents have in some inventories (that usually corresponds to the same object) the same largest- and second-largest-weight words and in such a case the plotted symbols overlap.

Homonymy appears when a word can be associated with more than one object and synonymy when an object can be associated with several words. However, the rules of our model contain probabilistic factors and so the definition of homonymy and synonymy must take this fact into account. We define homonymy as a word that with a relatively large probability can be associated with several objects. Typically such a situation occurs when a word uttered by the speaker appears in more than one inventory of the hearer as the largest-weight word. Consequently, the number of different largest-weight words is a measure of homonymy of the language (the smaller this number is, the more frequent the homonymy is). Analogously, synonymy most often occurs when speaker and hearer in their inventories corresponding to a certain object have the same largest- and second-largestweight words. In such a case, no matter which of them is selected for communication, it is quite probable that the meaning will be guessed correctly. Examples of such situations are shown in Fig. 4.

Since homonymy typically occurs when more than one inventory has the same largest-weight word, we examined in more detail the number of different largest-weight words and the results are shown in Fig. 5. One can notice that as the interval *r* from which the words are drawn increases, this number tends to the number of objects *n* and that means that homonyms become less frequent. This is because for large *r* there are many words to choose from and the probability that two inventories have the same largest-weight word decreases. Let us notice, however, that homonymy might appear also in some naming-game models with an unbounded reservoir of words (in our case it corresponds to the limit $r \rightarrow \infty$) [34].

Naively, one might expect that the number of different largest-weight words can be obtained from the simple probabilistic arguments: let us select randomly *n* numbers from the interval $\langle 1, r \rangle$ and check how many of them are different. We



FIG. 4. Homonymy occurs when a word (1244) can be associated with more than one object. Synonymy occurs when an object is associated with more than one word.

did such calculations and numerical results are also shown in Fig. 5 (small squares along the t=0 axis). One can notice that this agrees with simulations but only initially. The subsequent evolution of the model changes the initial distribution and the number of different largest-weight words increases in time. Since success rate and the number of different largest-weight words behave similarly [Figs. 2(a) and 2(b)], such a redistribution reduces homonymy and enables more efficient communication between agents. However, saturation below the maximal value (seen in Fig. 5), equal to the number of objects n, indicates that homonymy is a persistent feature of language.



FIG. 5. The time evolution of the number of different largestweight words for n=500, l=10, and $\epsilon=10^{-5}$. Small squares at the t=0 axis indicate the values for the randomly drawn words (see text). One can notice that during simulations a redistribution of largest-weight words takes place and that reduces the number of homonyms in the language. However, the number of largest-weight words saturates below n and that shows that homonyms are a persistent feature of language. For large range r the (almost) homonymy-free language is obtained.



FIG. 6. The time evolution of the success rate of utterances with largest- and second-largest-weight words. Relatively large success rate of utterances with second-largest weight words indicates that more than one word can be associated with some objects; i.e., some words can be treated as synonyms. Increasing in time fluctuations of the second-largest weight data are due to poor statistics caused by the decreasing number of such utterances (i.e., synonymy decreases over time). Simulations were made for n=500, $r=10^3$, l=10, and $\epsilon=10^{-5}$.

Figure 2(c) shows that a fraction of communication attempts is made with the second-largest weight words. When such an attempt is successful it usually means that there is more than one word that is associated with a given object, which for our purposes defines synonymy. That such words do ensure a relatively large success rate is confirmed in Fig. 6, where the time evolution of the success rate of utterances with largest- and second-largest-weight words is shown. Indeed, relatively large success rate of utterances with secondlargest weight words indicates that more than one word can be associated with some objects; i.e., some words can be treated as synonymous. However, the decrease in frequency of second-largest-weight utterances seen in Fig. 2(c) and (related with that) large fluctuations seen in Fig. 6 show that the role of synonyms diminish in time. In the long-time limit synonymous second-largest-weight words become irrelevant since entire communication proceeds with largest-weight words only.

A trace of synonymy can also be seen in Fig. 3. Indeed, overlapping plotting symbols (circles and crosses) show that both agents have a substantial fraction of the same largestand second-largest-weight words in corresponding inventories. This plot, however, does not tell us that for many of these pairs, the weight of the largest-weight word is so much dominant that other words from this inventory are essentially negligible (since they are never used), especially after long simulations. Although quantitative estimation of the role of homonymy and synonymy depends on parameters, some generic behavior seems to characterize our model. In particular, homonymy, although rare for large r, is a persistent feature of the language: except for the initial time interval, frequency of homonymous utterances remains constant. On the other hand, the frequency of synonymous utterances decreases in time.



FIG. 7. The average number of intervals N(d) of a given distance *d* between neighboring largest-weight words. Calculations were made for n=500, $r=10^3$, l=10, and $\epsilon=10^{-5}$. These data show that noise greatly magnifies redistribution of words so that they are more evenly distributed within the available range (overlaps and large distances between words are much less likely, comparing to the distribution where they are selected independently).

Provided that the model bears some similarity to the evolution of natural languages, one can expect that in presentday languages, that correspond to the long-time limit of the language that emerge in our model, synonymy, in agreement with some observations, would be rare. It was already suggested by Hurford [20] that rareness of synonymy is caused by the asymmetry of evolutionary benefits between speaker and hearer. Let us emphasize that our model uses only cultural (single-generation) mechanisms for the evolution of language. The results thus show that understanding of some basic features of homonymy and synonymy can be obtained within a much simpler model that does not take into account any evolutionary effects. Let us also notice that persistent homonymy and transient synonymy was also reported by Puglisi *et al.* [24] in an interesting model of category forma-



tion. In their model, which also includes cultural dynamics only, agents are exposed to the continuous environment (i.e., with an infinite number of objects) and such a behavior was observed even when the reservoir of possible words is unbounded. As a matter of fact transient nature of synonymy is a more generic property of the naming game, although in some models persistent synonymy was reported [35].

C. Effect of noise and distribution of words

All calculations reported so far were made for the noiseless case (p=0), i.e., under the assumption that communication of a word to another agent is perfect and cannot change the word. Now we relax this assumption and examine the role of noise that might distort the communicated word as specified in Eq. (2). In our opinion, especially at early stages of the evolution of language communication could be exposed to such a disturbance.

Because of noise, the received word might be different than that uttered by the speaker. If the difference is small, the hearer might still correctly decode it. We expect that this will be often the case when the amplitude of noise is small or the largest-weight words are well separated so that the small change does not lead to the overlap with some similar words. As we have already noticed (Fig. 5), during the evolution of the model a redistribution of largest-weight words takes place, which reduces homonymy and improves communication between agents. Figure 7 shows that noise greatly magnifies such a redistribution. In this figure we present the distribution of distances d between neighboring largest-weight words compared with the distribution where largest-weight words are selected randomly. One can notice that noise leads to the more even distribution (within the available range) with substantially reduced number of overlaps (d=0) as well as of large voids.

Noise also changes the distribution of second-largestweight words. Accumulation of points along the diagonal line seen in Fig. 8 shows that in presence of noise the second largest-weight words are very often close to the largestweight words. In such a case they should not be considered as synonyms (that are usually much different) but as the same words but, e.g., with a slightly modified pronunciation. When noise is absent there is no such accumulation (Fig. 3).

It is possible that noise played an important role in the evolution of language and helped to redistribute words within available phonetic space (Fig. 7) and/or reduced the number of synonyms (Fig. 8). Actually, it would be interesting to obtain the analog of the distribution of distances between words shown in Fig. 7, but obtained for natural languages. Although the very definition of distance between words remains under debate, various algorithms of mainly phonetic comparison are already in use [36] and some statistical analysis in principle could be made.

IV. CONCLUSIONS

FIG. 8. The distribution of largest-weight words and the secondlargest-weight words after $t=10^3$ simulations. Calculations were made for n=500, $r=10^3$, l=10, $\epsilon=10^{-5}$, and $\eta=0.05$. Different plotting symbols correspond to different agents.

In the present paper we studied an *n*-object naming game between two agents and examined the structure of the emergent common vocabulary. Our results show that after an initial transient a linguistic synchronization is reached and efficient communication of agents is established: speaker selects an object and a corresponding word that is communicated to hearer that usually correctly decodes the intended meaning. Our main results are twofold:

(i) A small fraction of communication attempts use homonyms or synonyms. Although homonyms reduce the efficiency of communication they appear to be a rather persistent feature of the language. On the other hand, synonyms do not reduce such an efficiency but are gradually expelled from the language. The model supports, thus, the observation that nowadays in natural languages synonyms are rare, but related observations were also made in another type of models by Puglisi *et al.* [24]. Moreover, it seems to us that the present model, that has only one generation of agents and does not refer to the notion of fitness, is simpler than that used by Hurford [20] where the rareness of synonymy was attributed to the asymmetry of payoff between speaker and hearer.

(ii) The second main result is to show that noise plays (or played) an important role in the evolution of language. It enhanced redistribution of words and probably contributed to the reduction in synonymy of the language.

It would be desirable to extend our model to a multiagent. Let us notice, however, that such simulations are likely to be computationally very demanding (in such a case the dynamics of the model will be slower and the amount of calculations needed for the model to reach the linguistic synchronization will be much larger). An additional problem might be related to examining the structure of emerging language and the present paper shows that even the preparatory (twoagent) version provides rich and nontrivial behavior. Since, however, human linguistic interactions take place in a multiagent regime, such an extension should be examined. Another possibility might be to introduce a fitness function and implement evolutionary changes that in some versions of naming-game models are known to result in qualitatively novel behavior [26]. Let us notice that our model neglects, among others, sound-merging effects as well as interactions of a given language with spatially neighboring languages. Such factors often provide an important source of homonyms and synonyms. These factors might be taken into account in a multiagent version of our model. Moreover, it would be interesting to examine a situation where the number of objects n could differ from the number of inventories, would depend on an agent, and in addition would be determined in some dynamical process of category formation as in the paper of Puglisi et al. [24].

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